N^* Regge Pole and Pion-Nucleon (3,3) Phase Shifts

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The consequence of the hypothesis that the N^* lies on a Regge trajectory for the low-energy pion–nucleon phase shifts in the $J=\frac{3}{2}$, $T=\frac{3}{2}$ state is investigated using the Khuri representation.

THE consequence of the hypothesis of Regge poles on the low-energy behavior of a scattering amplitude has recently been pointed out by Khuri.[1] The Khuri representation has subsequently been used by Khuri and Udgaonkar[2] to calculate the pion-nucleon phase shifts in the $T=\frac{1}{2}$, $P_{\frac{1}{2}}$ state on the basis of a model in which the scattering process proceeds through a single Regge-pole nucleon intermediate state in the direct channel. The case of neutron-proton scattering proceeding via a Regge-pole deuteron has been discussed elsewhere.[3] The purpose of this note is to extend the method of Ref. 2 to calculate the pion-nucleon phase shifts in the $T=\frac{3}{2}$, $P_{\frac{3}{2}}$ state. We consider the scattering process to proceed via a N^* isobar [pion-nucleon (3,3) resonance] intermediate state and treat the isobar as a Regge particle with a variable spin.

Using the Khuri representation, the contribution of the N^* Regge pole to the pion-nucleon $T=\frac{3}{2}$, $P_{\frac{3}{2}}$ partial-wave amplitude can be calculated in a straightforward manner. The result is[4]

$$a_-(\tfrac{3}{2},W)=-\tfrac{1}{2}\frac{\beta(W)}{\alpha(W)-\frac{3}{2}}\times[\exp(\alpha-\tfrac{3}{2})\xi_1+\exp(\alpha-\tfrac{3}{2})\xi_2]. \quad (1)$$

In (1), $\alpha(W)$ is the N^* Regge trajectory and $\beta(W)$ the corresponding residuum. W is the total c.m. energy. ξ_1 and ξ_2 are given by

$$\xi_1=\ln\left\{1+\frac{2}{k^2}+\left[\left(1+\frac{2}{k^2}\right)^2-1\right]^{\frac{1}{2}}\right\},$$

$$\xi_2=\ln\left\{\frac{W^2-m^2-2}{2k^2}-1+\left[\left(\frac{W^2-m^2-2}{2k^2}-1\right)^2-1\right]^{\frac{1}{2}}\right\}. \quad (2)$$

In (2), k is the c. m. momentum and m the nucleon mass. The pion mass has been set equal to unity. Following Khuri and Udgaonkar,[2] we next approximate Eq. (1) based on considerations of the threshold behavior of $\beta(k^2)$. The latter is given by

$$\beta(k^2)\simeq[k^2]^{\alpha_0-\frac{1}{2}} \quad \text{as} \quad k^2\to 0;\quad \alpha_0=\alpha(W)|_{k^2=0}, \quad (3)$$

while the terms involving the exponential behave as

$$\begin{aligned}&\exp(\alpha-\tfrac{1}{2})\xi_1\simeq[\tfrac{1}{4}k^2]^{-(\alpha_0-\frac{1}{2})}\\ &\exp(\alpha-\tfrac{1}{2})\xi_2\simeq[k^2/(2m-1)]^{-(\alpha_0-\frac{1}{2})},\quad \text{as}\quad k^2\to 0,\end{aligned} \quad (4)$$

so that the product $\beta\exp(\alpha-\frac{1}{2})\xi_1$ will be slowly varying and essentially real near threshold. We approximate it by a real constant C for the entire energy range under consideration

$$\beta\exp(\alpha-\tfrac{1}{2})\xi_1=C. \quad (5)$$

We next consider the form of the N^* trajectory $\alpha(W)$. The real part of $\alpha(W)$ may be written in the form

$$\text{Re}\alpha(W)=\tfrac{3}{2}+\epsilon(W-m^*). \quad (6)$$

In (6) m^* denotes the mass of the N^* isobar and ϵ the slope of the N^* trajectory. For the imaginary part of $\alpha(W)$ we assume the form

$$\text{Im}\alpha(W)=C_1[W-(m+1)]^{\frac{3}{2}}. \quad (7)$$

The form (7) for $\text{Im}\alpha(W)$ satisfies the requirement that $\alpha(W)$ be purely real below threshold. It is also consistent with the requirement of the correct threshold behavior[5] of $\text{Im}\alpha(W)$, viz.,

$$\text{Im}\alpha\simeq k^{2\alpha_0+1},\quad k^2\to 0. \quad (8)$$

The constant C_1, occurring in (7), may be expressed in terms of the width Γ of the (3,3) resonance using the relation

$$\Gamma=\frac{1}{m^*}\left[\text{Im}\alpha(W)\Big/\frac{d}{dW^2}\text{Re}\alpha(W)\right]_{W=m^*}. \quad (9)$$

From (6), (7), and (9), we obtain for the trajectory

$$\alpha(W)=\tfrac{3}{2}+\epsilon(W-m^*)+\tfrac{1}{2}i\Gamma\epsilon\left[\frac{W-(m+1)}{m^*-(m+1)}\right]^{\frac{3}{2}}. \quad (10)$$

The partial-wave amplitude $a_-(\frac{3}{2},W)$ finally becomes,

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[1] N. N. Khuri, Phys. Rev. **130**, 429 (1963). Khuri's formula was independently obtained by C. E. Jones, University of California Lawrence Radiation Laboratory Report UCRL-10700 (unpublished).

[2] N. N. Khuri and B. M. Udgaonkar, Phys. Rev. Letters **10**, 172 (1963).

[3] S. K. Bose and M. DerSarkissian, Nuovo Cimento (to be published).

[4] We are following the notation of V. Singh [Phys. Rev. **129**, 1889 (1963)]. The odd J-parity amplitude $a_-^0(J,W)$ interpolates to $a_-(\frac{3}{2},W)$. We have assumed that the former amplitude has no other singularity in the complex J plane except for a pole corresponding to N^*.

[5] A. O. Barut and D. E. Zwanziger, Phys. Rev. **127**, 974 (1962).

from (1) and (10),

$$a_-(\tfrac{3}{2},W)=-\frac{C}{2\epsilon}\left\{\frac{\exp(-\xi_1)+\exp(-\xi_2)\exp[(\xi_2-\xi_1)(\alpha(W)-\frac{1}{2})]}{W-m^*+\frac{1}{2}i\Gamma\left(\dfrac{W-(m+1)}{m^*-(m+1)}\right)^{\frac{3}{2}}}\right\}. \tag{11}$$

If we now specify the values of the parameters m^* and Γ from experiment, i.e., $m^*=8.91$, $\Gamma=0.72$, and estimate ϵ from the observed location of the next higher resonance, viz., the pion-nucleon resonance with $J=\frac{7}{2}^+$ at[6] 1920 MeV (ϵ turns out to be 0.41), the trajectory $\alpha(W)$ then is completely fixed. The (3,3) phase shift which is related to the partial-wave amplitude through the relation

$$a_-(\tfrac{3}{2},W)=(e^{i\delta_{33}}/k)\sin\delta_{33} \tag{12}$$

can now be calculated from (11). It is not necessary to known the value of the over-all multiplicative constant C appearing in (11) as the expression for phase shift $\delta_{33}=\tan^{-1}[\mathrm{Im}a_-(W)/\mathrm{Re}a_-(W)]$ is independent of the latter. The situation here is quite different from that in Ref. 2, where it was essential to determine the constant C (which was done by an extrapolation procedure). The difference between our case and that of Ref. 2 arises because, in the latter, the imaginary part of the (nucleon) trajectory was explicitly neglected. However, once the phase shifts are determined the amplitude $a_-(\frac{3}{2},W)$ is completely fixed and the constant C can then be determined by comparing the amplitude $a_-(\frac{3}{2},W)$ calculated at a fixed energy using (12), with that given by (11). This can, in particular, be done at the resonance energy $W=m^*$. The value of C, thus determined, is $C\simeq 0.3$.

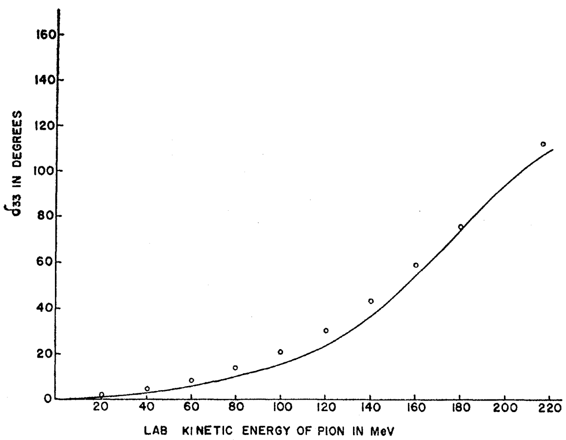

FIG. 1. Plot of the δ_{33} phase shifts in degrees as a function of the kinetic energy in MeV of the incident pion in the laboratory. Experimental points are denoted by circles.

The (3,3) phase shifts calculated from (10) are plotted in Fig. 1. As can be seen from the figure, our result for the energy dependence of δ_{33} is in good agreement with the experimental data.

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Note added in proof. A closely analogous approach to the present problem has been made by DerSarkissian [M. DerSarkissian. Louisiana State University, 1963 (unpublished)].

[6] We have assumed the spin of πN resonance at 1920 MeV to be $\frac{7}{2}^+$. We understand that this point is not yet completely settled, although the presence of a large $\cos^6\theta$ term in the pion angular distribution strongly favors this assignment. This assignment of spin $\frac{7}{2}^+$ has also been suggested by Glashow and Rosenfeld [S. L. Glashow and A. H. Rosenfeld, Phys. Rev. Letters **10**, 192 (1963)].